Hara Theorem, GIM Model, and Asymmetry in Weak Radiative Hyperon Decays

Elena N. Bukina[a], Vladimir M. Dubovik[a] and Valery S. Zamiralov[b]

[a] *Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia*

[b] *D.V. Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, Russia*

Abstract

It is shown that in the framework of GIM model, the parity-violating (PV) $\Sigma^+(uus) \Rightarrow p(uud) + \gamma$ decay amplitude does not vanish but just transforms into another PV decay amplitude, namely, of the $\Omega^+_{cc}(ccs) \rightarrow \Xi^+_{cc}(ccd) + \gamma$ decay. The known result is used that the relevant part of the CP-invariant effective current× current Hamiltonian changes sign under simultaneous quark changes $d \leftrightarrow s$ and $u \leftrightarrow c$. So, asymmetry of $\Sigma^+ \Rightarrow p + \gamma$ decay should not vanish in contrast to the Hara result of the old-fashioned $SU(3)_f$ model. At the same time, the zero asymmetry prediction persists for the $\Xi^- \Rightarrow \Sigma^- + \gamma$ decay.

INTRODUCTION

Weak radiative decays of hyperons were first analyzed theoretically forty years ago [1]- [2]. In 1964, in the framework of the unitary symmetry model, a theorem was proved by Hara that decay asymmetry in charged hyperon weak radiative decays $\Sigma^+ \Rightarrow p + \gamma$ and $\Xi^- \Rightarrow \Sigma^- + \gamma$ should vanish in the limit of exact $SU(3)_f$ [3]. Experimental discovery of a large negative asymmetry in the radiative decay $\Sigma^+ \Rightarrow p + \gamma$ [4], confirmed later [5], stimulated efforts to explain a drastic contradiction between experimental results and the Hara zero asymmetry prediction (see, e.g., [6] and references therein). The problem is actual up to now as shown by appearance of new approaches to cite among the most recent ones [7].

Another problem is related to inconsistency between $SU(3)_f$ symmetry [3] and quark model predictions [8] for the asymmetry value in hyperon weak radiative decays. Quark

models, while more or less succeeding in describing experimental data on branching ratios and asymmetry parameters (see, e.g., a review [6] and references therein), did not reproduce the Hara claim in the $SU(3)_f$ symmetry limit.

The origin of this discrepancy is under discussion up to now although many authors investigated this problem thoroughly [8]- [11] (see also [6] for a very complete list of publications). Recently, it was a subject of discussion in [12] and [13]. Both sides with fine arguments proved once more that the inconsistency persists though the authors have different points of view as to the origin of it. In [11] and [13], it was argued that while the covariant amplitude in [8] was taken gauge-invariant, the nonrelativistic reduction performed there proved to be gauge-variant. Instead, in [12], it was stated that gauge invariance was preserved in [8], and the origin of discrepancy should be hidden in other rather implicit assumptions as that of a sufficiently localized current.

On the other hand we showed that the problem may be solved in part by envoking the toroid dipole moment [14], although hardly it is possible to attain experimentally observed value.

We would like to show that the origin of both the problems can be in the specific formulation of basic assumptions on weak interaction taken in [3] in the framework of the $SU(3)_f$ symmetry model, and a remedy to the problems can be found already by taking a four-flavor model.

I. THE BASIC HARA RESULT

To describe strangeness-changing weak radiative hyperon decays of the baryon octet $B \to B' + \gamma$, in [3], usual basic assumptions as to the character of weak interactions were used :

(1) The effective weak interaction Hamiltonian is a sum of products of charged weak currents and their Hermitian conjugates.

(2) The weak interaction is CP-invariant. (Known violation of the CP-invariance was

ignored .)

These assumptions were read in [3] as invariance of the effective strangeness-changing weak interaction Hamiltonian under exchange of the unitary indices $2 \leftrightarrow 3$ (see also [15]):

$$H^{eff}_{SU(3)_f}(|\Delta S| = 1) = \frac{G_F}{2\sqrt{2}} sin\theta_C cos\theta_C \left\{[J^2_1, J^1_3]_+ + (2 \leftrightarrow 3)\right\},$$

where J^2_1 and J^3_1 are weak hadron currents with $|\Delta S| = 0$ and $|\Delta S| = 1$, respectively, in the usual $SU(3)_f$ tensor notation. Let us rewrite it in terms of the weak quark currents. The effective CP-invariant quark weak Hamiltonian with $|\Delta S| = 1$ in the three-quark model is

$$H^{eff}_W(|\Delta S| = 1) = \frac{G_F}{\sqrt{2}} sin\theta_C cos\theta_C \left\{(\bar{u}O_\mu d)(\bar{s}O_\mu u) + (\bar{u}O_\mu s)(\bar{d}O_\mu u)\right\}, \tag{1}$$

where $O_\mu = \gamma_\mu(1 - \gamma_5)$. The $H^{eff}_W(|\Delta S| = 1)$ is invariant under exchange of s- and d-quarks, $s \leftrightarrow d$ (or equivalently, under exchange of indices $2 \leftrightarrow 3$ because of the relation $B^\alpha_\beta = \epsilon_{\beta\gamma\delta}\{q^\beta, q^\gamma\}q^\delta$, where $\alpha, \beta, \gamma, \delta = 1, 2, 3$ and $u = q^1, d = q^2, s = q^3$, between the baryon wave functions and quark ones.)

The parity-violating (PV) parts of the amplitudes of weak radiative hyperon decays were written in [3] as follows:

$$\begin{aligned} M^{PV}_{SU(3)_f} = J^{(d)}_\mu \epsilon_\mu + H.C. = \\ \left\{a^d(\overline{B}^3_2 O^d_\mu B^1_1 - \overline{B}^1_1 O^d_\mu B^2_3) + b^d(\overline{B}^3_1 O^d_\mu B^1_2 - \overline{B}^2_1 O^d_\mu B^1_3) + \right. \\ \left. c^d(\overline{B}^1_2 O^d_\mu B^3_1 - \overline{B}^1_3 O^d_\mu B^2_1) + d^d(\overline{B}^1_1 O^d_\mu B^3_2 - \overline{B}^2_3 O^d_\mu B^1_1)\right\} \epsilon_\mu, \end{aligned} \tag{2}$$

with the gauge-invariant Lorentz structure $O^d_\mu = i\sigma_{\mu\nu}k_\nu\gamma_5$ [3] , ϵ_μ being the photon polarization 4-vector. The upper script d stays for *dipole transition moment*. To preserve invariance under exchange $2 \leftrightarrow 3$, Hara put $a^d = -d^d$, opening a possibility of the nonzero asymmetries for $(\Sigma^0, \Lambda) \to n + \gamma$ and $\Xi^0 \to (\Sigma^0, \Lambda) + \gamma$ decays. At the quark level, a requirement of the CP-invariance in the form $d \leftrightarrow s$ just prescribes that amplitudes of the decays $(\Sigma^0, \Lambda)(usd) \to n(ddu) + \gamma$ (+its HC) transform into the HC amplitudes of quite different decays $\Xi^0(ssu) \to (\Sigma^0, \Lambda)(uds) + \gamma$.

Instead of this, PV transition amplitudes of the decays $\Sigma^+ \to p + \gamma$ and $\Xi^- \to \Sigma^- + \gamma$ disappear in Eq.(2). The relevant terms $\overline{B}^3_1 O^d_\mu B^1_2$ and $\overline{B}^1_2 O^d_\mu B^3_1$, which should be invariant

under exchange of indices $2 \leftrightarrow 3$, change signs under Hermitian conjugation; therefore, the condition $b^d = c^d = 0$ should be imposed. The origin of this result is readily seen at the quark level. Under the exchange $d \leftrightarrow s$, the PV amplitudes of decays $\Sigma^+(uus) \to p(uud)+\gamma$ and $\Xi^-(ssd) \to \Sigma^-(dds)+\gamma$ are transformed into the respective HC amplitudes of the same decays but with a wrong sign.

This is in fact a source of all the troubles with the hyperon weak radiative decays in the framework of the $SU(3)_f$ model, as it just gives zero asymmetry for $\Sigma^+ \to p+\gamma$ and $\Xi^- \to \Sigma^- + \gamma$ decays.

II. GIM MODEL AND PV AMPLITUDES OF THE WEAK RADIATIVE HYPERON DECAYS

The simple picture of the weak interaction used in [3] was in fact based on a single weak isodublet $\begin{pmatrix} u \\ d_C \end{pmatrix}_L$, and led, as it is well known, to the existence of the strangeness- changing neutral weak current. This difficulty unknown in 1964 was overcome in the famous GIM model [16] where another weak isodublet $\begin{pmatrix} c \\ s_C \end{pmatrix}_L$, was introduced in order to make the neutral weak current diagonal in quark flavors. Following this we try to rewrite the Hara theorem in the context of the four-flavor scheme (for a moment not taking a six-flavor one) in order to get rid of the undesirable strangeness-changing neutral current implicitly hidden in [3].

In the framework of the GIM model, the relevant part of the CP-invariant effective Hamiltonian containing $|\Delta S| = 1$ piece reads now in terms of quarks as

$$H^{eff}_{GIM} = \frac{G_F}{\sqrt{2}} sin\theta_C cos\theta_C \Big\{ (\bar{u}O_\mu d + \bar{c}O_\mu s)(\bar{s}O_\mu u - \bar{d}O_\mu c) + \\ (\bar{u}O_\mu s - \bar{c}O_\mu d)(\bar{d}O_\mu u + \bar{s}O_\mu c) \Big\} . \quad (3)$$

But the Hamiltonian H^{eff}_{GIM} is no longer invariant under either change $s \leftrightarrow d$ or $c \leftrightarrow u$. Instead of this under a simultaneous change $s \leftrightarrow d$ and $c \leftrightarrow u$, it just changes an overall sign. We try now to insert this Ansatz into the effective flavor-changing electromagnetic current.

And this turns to be a solution of the Hara puzzle.

Really, now for the $\Sigma^+ \to p+\gamma$ PV decay amplitude , the Hermitian conjugation and invariance of H^{eff}_{GIM} given by Eq.(3) under the flavor exchange are uncorrelated.

To see this in terms of $SU(4)_f$ baryon wave functions, along the lines of the Hara proof [3] and a discussion in [15], we construct an appropriate baryon $SU(4)_f$ weak radiative transition current that

(i) conserves built-in weak interaction properties (1) and (2);

(ii) mantains gauge invariance .

The corresponding matrix element should be invariant under an overall change of sign with simultaneous changes of indices $1 \leftrightarrow 4$ and $2 \leftrightarrow 3$. This statement is essentially similar to that made by Hara, [3] and independent of the conjecture of the U- or P-spin used in [17]. In this way, (i) would be satisfied at the level of the four-flavor model, and instead of Eq.(2), we obtain:

$$\begin{aligned}
M^{PV}_{GIM}(|\Delta S|=1) = J^{PV}_\mu \epsilon_\mu + H.C. = \\
\Big\{ a^{pv}(\overline{B}^{34}_2 O^d_\mu B^1_{14} - \overline{B}^{14}_1 O^d_\mu B^2_{34} - \overline{B}^{21}_3 O^d_\mu B^4_{41} + \overline{B}^{41}_4 O^d_\mu B^3_{21}) + \\
b^{pv}(\overline{B}^{34}_1 O^d_\mu B^1_{24} - \overline{B}^{24}_1 O^d_\mu B^1_{34} - \overline{B}^{21}_4 O^d_\mu B^4_{31} + \overline{B}^{31}_4 O^d_\mu B^4_{21}) + \\
c^{pv}(\overline{B}^{14}_2 O^d_\mu B^3_{14} - \overline{B}^{14}_3 O^d_\mu B^2_{14}) + \\
d^{pv}(\overline{B}^{14}_1 O^d_\mu B^3_{24} - \overline{B}^{24}_3 O^d_\mu B^1_{14} - \overline{B}^{14}_4 O^d_\mu B^2_{13} + \overline{B}^{13}_2 O^d_\mu B^4_{14}) + \\
f^{pv}(\overline{B}^{13}_1 O^d_\mu B^1_{12} - \overline{B}^{12}_1 O^d_\mu B^1_{13} + \overline{B}^{34}_4 O^d_\mu B^4_{24} - \overline{B}^{24}_4 O^d_\mu B^4_{34}) \Big\} \epsilon_\mu,
\end{aligned} \tag{4}$$

where $SU(4)$ 20-plet baryon wave functions can be written in terms of quark wave functions as usual

$$B^\alpha_{\beta\gamma} = \epsilon_{\beta\gamma\eta\rho}\{q^\alpha, q^\eta\} q^\rho, \quad \alpha,\beta,\gamma,\eta,\rho = 1,2,3,4$$

and $u=q^1, d=q^2, s=q^3, c=q^4$. We remind that $SU(3)$ octet baryons are

$$B^1_{34} = p, \quad B^1_{24} = \Sigma^+, \quad B^3_{14} = \Xi^-, \quad B^3_{24} = \Xi^0,$$

$$B^2_{14} = \Sigma^-, \quad B^1_{14} = \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0, \quad B^2_{34} = n,$$

while those relevant charmed ones are

$$B_{13}^4 = \Xi_{cc}^+(ccd), \quad B_{12}^4 = \Omega_{cc}^+(ccs),$$

$$B_{13}^2 = \Sigma_c^0(cdd), \quad B_{14}^4 = \frac{2}{\sqrt{6}}\Xi_{cs}^{\prime 0}(csd), \quad B_{12}^3 = \Omega_c^0(css)$$

$$B_{34}^4 = \frac{2}{\sqrt{6}}\Lambda_c^+(cud), \quad B_{13}^1 = \frac{1}{\sqrt{2}}\Sigma_c^+(cud) + \frac{1}{\sqrt{6}}\Lambda_c^+(cud),$$

$$B_{24}^4 = -\frac{2}{\sqrt{6}}\Xi_{cs}^+(csu), \quad B_{12}^1 = \frac{1}{\sqrt{2}}\Xi_{cs}^{\prime +}(csu) + \frac{1}{\sqrt{6}}\Xi_{cs}^+(csu).$$

The main point is that now no coefficient in Eq.(4) should be put equal to zero, so neither $\Sigma^+ \to p + \gamma$ nor $\Xi^- \to \Sigma^- + \gamma$ decay PV amplitudes vanish.

We repeat once more that, in the Hara formulation, the term $\overline{B}_1^3 O_\mu^d B_2^1$ in Eq.(2) (which describes PV part of the $\Sigma^+ \to p + \gamma$ decay) has, as its Hermitian conjugation term (HC), $-\overline{B}_1^2 O_\mu^d B_3^1$, while if the assumption (i) reads as invariance under change $2 \leftrightarrow 3$, there must be $+\overline{B}_1^2 O_\mu^d B_3^1$. This results in a zero contribution to the PV amplitude of the $\Sigma^+ \to p + \gamma$ decay . The same reasoning is valid also for the term describing $\Xi^- \to \Sigma^- + \gamma$ decay.

But now with the requirement (i) formulated in the form consistent with the GIM model, the term $(\overline{B}_1^{34} O_\mu^d B_{24}^1 - \overline{B}_1^{24} O_\mu^d B_{34}^1)$ corresponding to the $\Sigma^+(uus) \to p(uud) + \gamma$ decay (+ its HC) just transforms into the term $(\overline{B}_4^{31} O_\mu^d B_{21}^4 - \overline{B}_4^{21} O_\mu^d B_{31}^4)$ that describes the PV part of the $\Omega_{cc}^+(ccs) \to \Xi_{cc}^+(ccd) + \gamma$ decay amplitude (+ its HC).

Instead of this the term $(\overline{B}_2^{14} O_\mu^d B_{14}^3 - \overline{B}_3^{14} O_\mu^d B_{14}^2)$ describing the PV part of the $\Xi^-(ssd) \to \Sigma^-(dds) + \gamma$ decay (+ its HC) transforms into itself with the same sign. It is easy to see that the corresponding PC part of this amplitude vanishes, so a zero asymmetry prediction persists for this decay. But this result does not contradict quark model calculations where only the $s \to d + \gamma$ decay diagram contributes, and its PV part vanishes in the limit $m_s = m_d$ [17,18].

So, none of the PV amplitudes of the hyperon octet radiative decays should be zero, if general requirements like properties under Hermitian conjugation, CP-invariance, etc, are applied in the framework of the GIM model (not talking of the six-flavor scheme for a moment). Disregarding new baryons containing c-quark in Eq.(4) and omitting index 4

altogether, we formally arrive at Eq.(2). Also, $|\Delta S| = 1$ neutral weak current appears and with it inadmissible values for $B \to B' + e^+e^-$ rates. The unique mode to go to the $SU(3)$ symmetry model limit, without falling in troubles with weak neutral currents, would be to put the Cabibbo angle equal to zero, forbidding thus all the processes with $|\Delta S| = 1$ in the sector of quarks u,d,s.

III. CONCLUSION

So, the Hara prediction proves to be valid only in the old-fashioned $SU(3)_f$ model where by default there are also strangeness-changing neutral currents, about which almost nobody worried in 1964. Already in the framework of the GIM model, even not taking into account more quark flavors, the Hara prediction of the zero asymmetry in the $\Sigma^+ \to p + \gamma$ decay is no longer valid. It is interesting that due to vanishing of the parity-conserving amplitude, the zero asymmetry prediction persists for the $\Xi^- \to \Sigma^- + \gamma$ decay.

Thus, there is no contradiction of the unitary model approach with the quark model one either for the $\Sigma^+ \to p + \gamma$ decay or for the $\Xi^- \to \Sigma^- + \gamma$ one.

We conclude our letter with a remark that maybe experimental observation of the nonzero asymmetry in the $\Sigma^+ \to p + \gamma$ decay [4] could serve as indication of some serious difficulties in the description of electroweak interactions in the framework of the 3-quark model already in early seventies and could be just one more argument in favor of the 4th quark and diagonal flavor structure of weak neutral currents.

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